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Diffraction gratings and the plane mirror

By E. Hulthén and E. Lind

With 16 figures in the text

Introduction

This investigation arose from proposals set forth some time ago by one of the writers (E. H.) in a discussion about how to make use of the blaze-effect of modern gratings to gain high brilliancy and resolution in spectrum.

A simple way to increase the resolution of a light bundle, dispersed in a prism, is to reflect it in its opposite direction of entrance, thus dispersing twice during its passage through the prism. By the aid of autocollimation in lenses this principle has been widely applied in spectroscopic apparatus, generally known as the Littrow mounting. To some extent it is also applied to the plane and the concave gratings, although here the beam of light is only diffracted once by the grating. The strong blaze, which appears from gratings ruled with shaped diamonds, where a great part of the diffracted light is concentrated into one order (Ref. 1–5), makes it possible to adapt this method more completely. An obvious step is to place a plane mirror in the blaze-direction of the diffracted beam, reflecting it back to the grating for a second diffraction. It is even conceivable that a fraction of the beam, obeying the law of diffraction, will pass repeatedly between the grating and the mirror with increasing dispersion and resolution. The spectra produced in this way will here, according to the corresponding number of diffractions, be termed primary, secondary spectra and so on.

These ideas may be applied to the plane grating in autocollimation without any special devices. Their application to the concave grating is more intricate, but they give rise to some new grating mountings. To what extent the method of double or multiple diffraction of light may increase the spectroscopic resolution depends greatly on the optical qualities of the grating and the mirror. It will, however, be immediately realized that brilliancy may be gained by operating the grating repeatedly in an order of blaze, instead of operating it directly in the corresponding higher order of the spectrum. The high orders of spectrum are generally also more troubled by ghosts due to imperfections in the grating.

Unfortunately, the gratings at our disposal for this investigation almost lack that blaze which is so beautifully demonstrated by some new gratings and with which our method should have its best application. Its usefulness was also at first disturbed by the superposition of different combinatory orders in spectrum, so that e.g. the secondary spectrum of the 4th order will be superposed by the combinatory orders (1 + 3) or (3 + 1), which appear when the mirror is placed within a certain distance from the plane grating. A good deal of attention has been paid to the analysis of these complications (§§ 2, 4). Nevertheless, we find the composition with grating–plane mirror deserving of attention, especially when applied to the concave grating.

Our suggestions regarding the mountings of gratings and mirrors will be discussed in the following order.

§ 1. The plane grating with its normal directed toward the slit.
§ 2. Superposition of spectra due to rays of oblique reflections at the mirror according to § 1.
§ 3. The plane grating with its normal directed toward the mirror.
§ 4. Superposition of spectra due to oblique rays of reflections at the mirror according to § 3.
§ 5. The concave grating and the plane mirror in non-parallel rays of light.
§ 6. The concave grating and the plane mirror in parallel rays of light and by stigmatic configuration.
§ 7. The concave grating and the plane mirror in parallel rays of light and by astigmatic configuration.
§ 8. The grating and the mirror in parallel rays of light at equal angles of incidence and diffraction.

§ 1. The plane grating with its normal directed toward the slit

A plane grating G, with its normal directed toward the slit S, is in fig. 1 mounted according to the principle of autocollimation. A parallel beam of light through the achromatic lens L, diffracted by the grating at an angle β, will obey the condition:

$$m\lambda = \sigma \sin \beta, \tag{1 a}$$

where m is the order of spectrum and σ the grating constant.

The diffracted and parallel beam of wave-length λ is reflected normally by the plane mirror M toward the grating, and once more diffracted it will partly

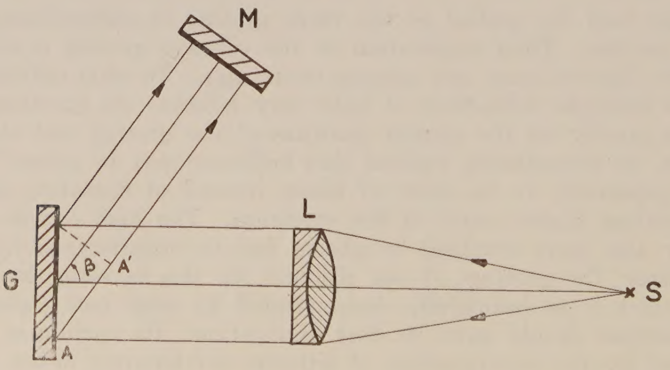

Fig. 1.

form a final image at the slit, partly be spread in other directions. For the beam focussed at the slit we have the equation

$$m\lambda = \sigma \sin \beta, \tag{1 b}$$

where β is now the angle of incidence. From (1 a) and (1 b) we see that the secondary spectrum at the slit is of the order $2m$.

In order to deduce an expression for the dispersion of this spectrum we consider the passage of the light in two stages according to fig. 2:

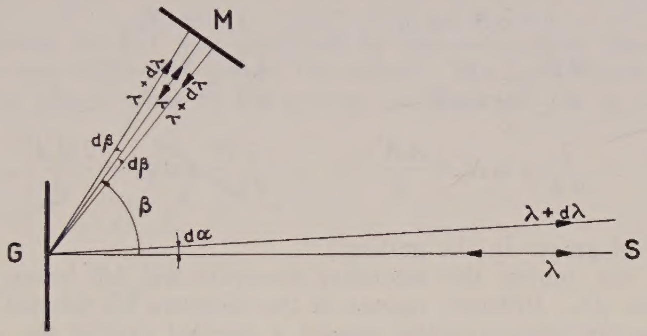

Fig. 2.

slit–grating–mirror,

$$m\lambda = \sigma(\sin 0 + \sin \beta),$$

$$m(\lambda + d\lambda) = \sigma[\sin 0 + \sin(\beta + d\beta)],$$

mirror–grating–slit,

$$m\lambda = \sigma(\sin \beta + \sin 0),$$

$$m(\lambda + d\lambda) = \sigma[\sin(\beta - d\beta) + \sin d\alpha].$$

Correspondingly, we get by subtraction

$$m\,d\lambda = \sigma \cos \beta\, d\beta,$$

$$m\,d\lambda = -\sigma \cos \beta\, d\beta + \sigma\, d\alpha.$$

This gives for the angular dispersion of the secondary spectrum

$$\frac{d\alpha}{d\lambda} = \frac{2m}{\sigma}. \tag{1 c}$$

Before reflection against the mirror the angular dispersion of the diffracted beam was

$$\frac{d\beta}{d\lambda} = \frac{m}{\sigma \cos \beta}. \tag{1 d}$$

It thus appears that the angular dispersion of the secondary spectrum is greater than that of a single diffraction, as long as $\beta < 60^\circ$. This holds also for the linear dispersion, as this is proportional to the angular dispersion.

Regarding the resolution of the secondary spectrum at the slit this is directly expressed by the difference in the paths of the bordering rays, expressed in wave number units. Thus:

$$\frac{\lambda}{d\lambda} = \frac{2\,AA'}{\lambda}. \tag{1 e}$$

This expression, which will be used repeatedly in this paper, can be shown to be in accordance with the general expression for the resolution of spectroscopic configurations, $\frac{\lambda}{d\lambda} = s\frac{d\beta}{d\lambda}$, where s is the linear aperture of the beam, as follows:

$$\begin{array}{ll}
\text{slit–grating–mirror,} & \text{mirror–grating–slit,} \\
s = \sigma N \cos\beta, & s = \sigma \cdot N, \\
\frac{d\beta}{d\lambda} = \frac{m}{\sigma\cos\beta}, & \frac{d\beta}{d\lambda} = \frac{2m}{\sigma}, \\
\therefore \frac{\lambda}{d\lambda} = mN = \frac{AA'}{\lambda}, & \therefore \frac{\lambda}{d\lambda} = 2mN = \frac{2\,AA'}{\lambda}.
\end{array}$$

(N = number of groves in the grating.)

By tilting the mirror the secondary spectrum will fall below, above or at the side of the slit. Different regions of the spectrum are selected by mounting the mirror on an arm, rotatable around a vertical axis in the centre line of the grating. The available region for the secondary spectrum around the slit is limited by the fact that rays reflected obliquely from the mirror will pass outside the grating. Therefore, if we increase the distance of the mirror to the grating, the available region for the secondary spectrum will decrease around the slit.

A fraction of the light, diffracted twice at the grating is reflected once more against the mirror. It will therefore return to the grating, producing a tertiary spectrum at the slit. Equations analogous to 1 a and 1 b show that this spectrum is of the order 4 m.

The secondary and tertiary images of the yellow doublet $\lambda\lambda$ 5770, 5791 and the green line λ 5461 of Hg are seen on the spectrograms in fig. 3. All spectrograms from the plane grating mountings are reproduced in natural sizes in order to verify the dispersion relations, deduced in §§ 1–4.

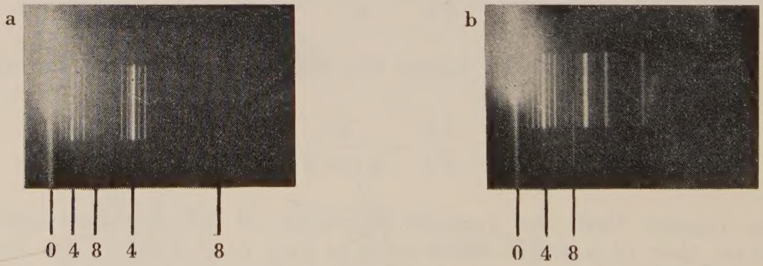

Fig. 3.

The secondary and tertiary spectra of the yellow Hg-doublet $\lambda\lambda$ 5770, 5791 (3 a) and of the green Hg-line λ 5461 (3 b), 4 and 8 corresponding to their 4th and 8th final orders, 0: slit image.

The plane grating used in this investigation was ruled on Rowland's engine in 1889. Its ruled surface is 82×50 mm^2 and $\sigma = 17\,350$ Å (14 440 lines to the inch). An aluminized slab of glass, 100×80 mm^2, was used as plane mirror. The grating was mounted in autocollimation by a plane-convex, achromatic lens of 165 cm focal length.

§ 2. Superposition of spectra due to rays of oblique reflections at the mirror according to § 1

The discussion in § 1 was simplified by the assumption that only rays of one order m were diffracted toward the mirror. This is no longer the case when the mirror is placed close to the grating, as illustrated in fig. 4. Obviously, final images of the order $2\,m$ can also be produced by rays of wave-length λ, at first diffracted in the order $m - 1$ and after reflection at the mirror diffracted in the order $m + 1$ and vice versa. Let us suppose that the final directions of these rays deviate by the angles ν and μ from the grating-normal. According to fig. 4, we have:

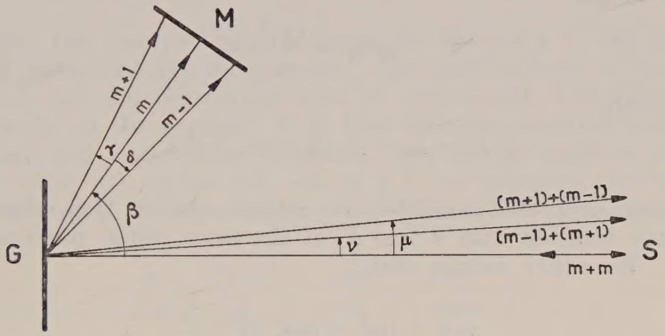

Fig. 4.

slit–grating–mirror,	mirror–grating–final direction,
$m\,\lambda = \sigma \sin \beta,$	$m\,\lambda = \sigma \sin \beta,$
$(m - 1)\,\lambda = \sigma \sin (\beta - \delta),$	$(m + 1)\,\lambda = \sigma\,[\sin (\beta + \delta) + \sin \nu],$
$(m + 1)\,\lambda = \sigma \sin (\beta + \gamma),$	$(m - 1)\,\lambda = \sigma\,[\sin (\beta - \gamma) + \sin \mu].$

Here $\gamma > \delta$ and therefore $\mu > \nu > 0$. Consequently the final images (spectral lines) of $2\,m\,\lambda$ will not coincide. Analogous expressions may be deduced for rays of the order combinations $(m \mp 2) + (m \pm 2)$ and so on, which all produce final images of the order 2 m.

Generally the case of coincidence between two spectral lines produced by a diffraction grating is met by the condition:

$$m\,\lambda = m'\,\lambda'. \tag{2 a}$$

In our case, this condition is more complicated. Let us now enquire whether there exist two lines of wave-lengths λ_1 and λ_2, produced by the order combinations $(m \mp 1) + (m \pm 1)$, which might coincide with $2m\lambda$ at the slit. The following relations will then hold for λ_1:

slit–grating–mirror,	mirror–grating–slit,
$(m-1)\lambda_1 = \sigma \sin(\beta - \delta_1)$,	$(m+1)\lambda_1 = \sigma \sin(\beta + \delta_1)$.

Therefore:

$$\frac{m-1}{m+1} = \frac{\sin(\beta-\delta_1)}{\sin(\beta+\delta_1)} \text{ or } \operatorname{tg}\delta_1 = \frac{\operatorname{tg}\beta}{m}.$$

Analogously we get:

$$\operatorname{tg}\delta_2 = \frac{\operatorname{tg}\beta}{m},$$

and consequently:

$$\lambda_1 = \lambda_2 = \lambda' \ (\delta_1 = \delta_2 = \delta').$$

When oblique rays are permitted to reflect against the mirror, λ of the order $2m$ may coincide with a line λ' of the same order, produced via $(m \mp 1)$ and $(m \pm 1)$. We easily deduce that

$$m\lambda = [m^2 + \tan^2\beta]^{1/2} \cdot \lambda'. \tag{2 b}$$

This will hold as long as we restrict ourselves to the order combinations $m + m$, $(m-1) + (m+1)$ and $(m+1) + (m-1)$. At the slit, however, further lines λ' may be produced by other combinations of orders too, and in general their final images are defined by the following conditions:

slit–grating–mirror,	mirror–grating–slit,
$p\lambda' = \sigma \sin(\beta - \delta')$,	$q\lambda' = \sigma \sin(\beta + \delta')$,

where $q \geqq p$. This gives:

$$2m\lambda = [(q+p)^2 + (q-p)^2 \tan^2\beta]^{1/2} \cdot \lambda'. \tag{2 c}$$

Obviously, (2 c) includes the relations (2 a) and (2 b).

By the aid of this relation and knowing the dimensions of the mirror and the grating (§ 1) it is a simple task to calculate the minimum distances between the mirror and the grating at which such confusing superpositions between λ and λ' will completely disappear. In table I some of these distances e are given in cm for the case $m = 2$ (i.e. final order 4) in our plane grating mounting. Obviously, normal high-order lines of λ, where $\delta' = 0$ and $p = q$, cannot be eliminated in this way.

Table I

β	10°		20°		30°		40°		50°	
λ	1500		3000		4350		5600		6660	
	λ'	e	λ'	e	λ'	e	λ'	e	λ'	e
$p=1,\ q=2$	1970	85	3880	41	5650	26	7060	18	8300	13
$p=1,\ q=3$	1500	56	2880	28	4220	18	5100	12	5800	8
$p=2,\ q=3$	1210	142	2390	69	3460	42	4470	30	5180	21
$p=3,\ q=4$	870	199	1700	96	2480	59	3300	42	4480	29

§ 3. The plane grating with its normal directed toward the mirror

In this case the beam of rays is similar to that of § 1, but the angles of incidence and diffraction are reciprocated. The parallel beam of rays is incident at an angle α, and monochromatic light of wave-length λ is diffracted in the order m normally to the grating. It is then reflected normally from the mirror to the grating and once more diffracted, now at the angle α, producing an image of the order $2m$ at the slit. As in § 1, we calculate the resolution and the angular dispersion of the secondary spectrum which are correspondingly

$$\frac{\lambda}{d\lambda} = \frac{2AA'}{\lambda}; \qquad \frac{d\alpha}{d\lambda} = \frac{2m}{\sigma\cos\alpha}. \tag{3 a}$$

From this it appears that the secondary spectrum conforms exactly with that of an ordinary Littrow mounting with $\alpha = \beta$. Nothing more can be gained in resolution and dispersion, however many times the beam may be reflected between the grating and the mirror. The real difference between the two types of spectrum consists, however, in the fact that the primary spectrum is produced by a grating acting directly in the order $2m$, while in the secondary

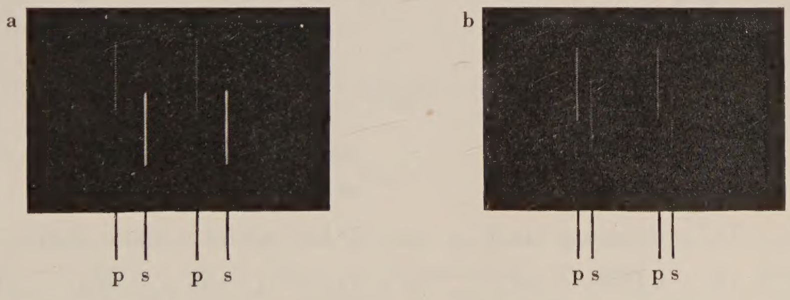

Fig. 5.

Primary (p) and secondary (s) spectra of the yellow Hg-doublet in the 4th order, 5 a showing the intensity effect produced by the blaze of the grating in one of its second orders and 5 b the lack hereof in its opposite second order.

spectrum the grating acts twice in the order m. This may entail a considerable gain of intensity in the second case, especially if the order m is distinguished by blaze. The spectrograms in fig. 5 give some idea of this effect, even if the "blaze" of our grating is of minor strength.

Abstracting at first from the complications which appear when the mirror is placed near the grating (§ 4), this mounting, like that of § 1, will produce only secondary spectra of even orders. The correct position of the mirror, parallel to the grating, is conveniently attained by turning and tilting it in such a way that the spectral lines of the secondary spectrum coincide with those of the primary spectrum, below or above the latter.

§ 4. Superposition of spectra due to rays of oblique reflections at the mirror, according to § 3

The spectra produced at the mounting described in the preceding paragraph are rather confused, owing to superposing spectra appearing when the mirror is placed close to the grating. The complications are similar to those, discussed in § 2 and here as there they will be explained as due to rays obliquely reflected at the mirror.

In A below we will discuss the coincidence between spectra of different order combinations, in B the appearance of uneven final orders in the secondary spectrum, and in C the effects, which arise when the mirror is not exactly parallel to the grating.

A. Suppose a monochromatic beam of wave-length λ diffracted in two stages at the grating according to fig. 6 as follows:

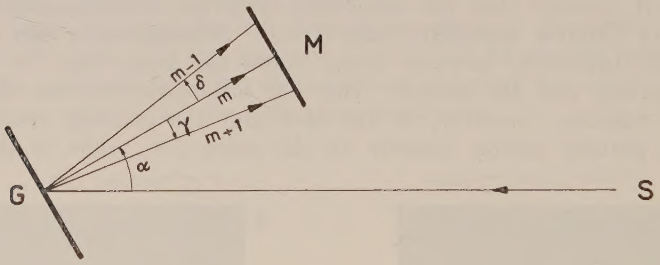

Fig. 6.

slit-grating-mirror,	mirror-grating-final image,
$m\lambda = \sigma \sin \alpha,$	$m\lambda = \sigma \sin \alpha,$
$(m-1)\lambda = \sigma(\sin \alpha - \sin \delta),$	$(m+1)\lambda = \sigma[\sin \delta + \sin(\alpha + \nu)],$
$(m+1)\lambda = \sigma(\sin \alpha + \sin \gamma),$	$(m-1)\lambda = \sigma[-\sin \gamma + \sin(\alpha + \mu)].$

From these relations we get $\gamma = \delta$ and $\mu = \nu = 0$, i.e. the final images of all three rays of the order $2m$ will coincide at the slit. Analogously, the rays of the order combinations $(m \mp 2) + (m \pm 2)$ etc. will contribute to this final image of $2m\lambda$, as soon as the mirror stands close to the grating. In addition to this, of course, the primary spectral line of $2m\lambda$ will also coincide with the final image, unless horizontally separated herefrom by the tilt of the mirror.

As the final image of $2m\lambda$ is thus produced from a number of different combinations of orders, it is of interest to know their dispersions. As an example we calculate the dispersion for the combination $(m-1)+(m+1)$ as follows:

$$(m-1)\,d\lambda = -\sigma\cos\delta\,d\delta, \qquad (\alpha \text{ const.})$$

$$(m+1)\,d\lambda = \sigma(\cos\delta\,d\delta + \cos\alpha\,d\alpha), \qquad (\alpha \text{ variable})$$

and consequently

$$\frac{d\alpha}{d\lambda} = \frac{2m}{\sigma\cos\alpha}.$$

Identical expressions are derived for all other combinations of the final order $2m$, irrespectively of whether the rays pass via the mirror or not (i.e. the primary spectrum). This is the case also if the final image is of an uneven order $2m+1$, and it is found to hold even if the final image does not coincide with the slit, all under the presumption that the mirror and the grating are exactly parallel. This fact may be applied to Echelette gratings where the spread of orders of the visible light is small and where some consecutive orders are often very brilliant.

B. The appearance of uneven orders in the secondary spectrum at first seemed rather confusing, as indicating the existence of half order diffractions emanating from the grating. The introduction of rays of oblique incidence to the mirror at once suggests a very simple explanation.

Consider a primary spectrum $(2m+1)\lambda$ at the slit. Then, according to fig. 7, $(2m+1)\lambda = 2\sigma\sin\alpha$.

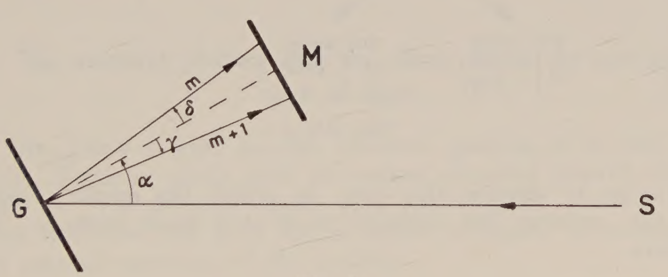

Fig. 7.

Then, for rays diffracted to the mirror, we have $m\lambda = \sigma(\sin\alpha - \sin\delta)$ and $(m+1)\lambda = \sigma(\sin\alpha + \sin\gamma)$, from which equations one gets $\gamma = \delta$. Now, considering the secondary spectra produced by these rays, we immediately find that they will coincide with the primary spectrum of the same order $2m+1$ at the slit, in accordance with the general statement in section A. Other combinations of the order $2m+1$, e.g. $(m-1)+(m+2)$, will give similar results.

This explains the appearance of secondary spectra of uneven orders, as well as the fact that the grating is thereby illuminated in vertical strips as in fig. 8 a and in accordance with the geometry of fig. 8 b. Fig. 8 c shows the primary and secondary spectra of the yellow Hg-doublet in the 3rd order.

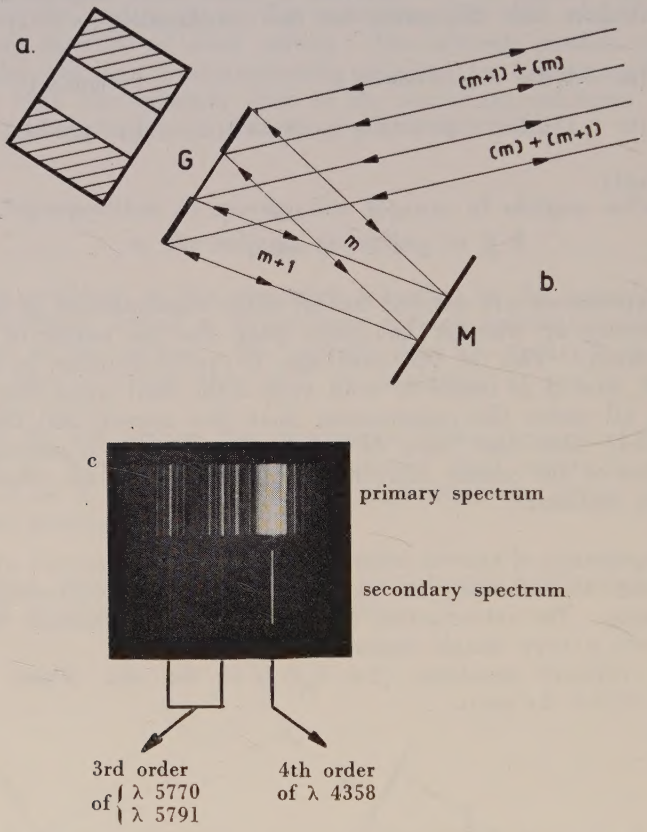

Fig. 8 a—c.

C. It remains to discuss the case, in which the mirror is not absolutely parallel to the grating, but inclined to it at a small angle φ. According to fig. 9, we have

$$\begin{array}{ll}
\text{slit–grating–mirror,} & \text{mirror–grating–final image,} \\
m\lambda = \sigma \sin \alpha, & m\lambda = \sigma [\sin 2\varphi + \sin(\alpha + \varepsilon)], \\
(m-1)\lambda = \sigma(\sin\alpha - \sin\gamma), & (m+1)\lambda = \sigma[\sin(2\varphi + \gamma) + \sin(\alpha + \nu)], \\
(m+1)\lambda = \sigma(\sin\alpha + \sin\gamma), & (m-1)\lambda = \sigma[\sin(2\varphi - \gamma) + \sin(\alpha + \mu)],
\end{array}$$

and thus

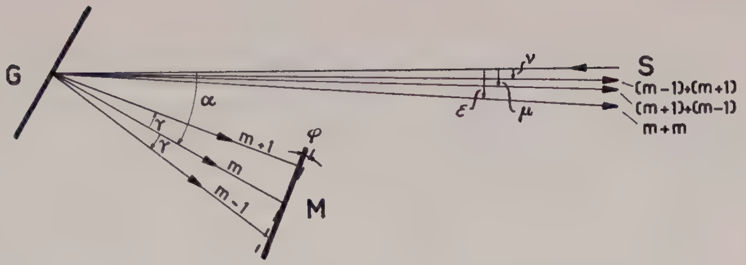

Fig. 9.

$$\sin(\alpha + \varepsilon) = \sin\alpha - \sin 2\varphi,$$
$$\sin(\alpha + \nu) = \sin\alpha - 2\sin\varphi\cos(\gamma + \varphi),$$
$$\sin(\alpha + \mu) = \sin\alpha - 2\sin\varphi\cos(\gamma - \varphi).$$

Consequently, the angles ε, ν and μ are different, and therefore the final image of $2m\lambda$ in the secondary spectrum is split into three components. Considering also the rays of $m \pm 2$ at the mirror, the final image of $2m\lambda$ will be split into five components and so on. Analogous calculations show that an uneven final image is split into two and four components respectively.

In consequence of this, distinct images of the secondary spectrum require the grating and its mirror to be perfectly parallel — as long as attention has to be paid to oblique rays of incidence at the mirror. For ordinary gratings giving large angle dispersion the whole grating will not be illuminated by these oblique rays and thus the purity of the spectrum decreases. It is therefore advisable in this case to withdraw the mirror so far from the grating that no oblique rays give any contributions to the secondary spectrum. As in § 2, these minimum distances can easily be estimated.

§ 5. The concave grating and the plane mirror in non-parallel rays of light

Applying the plane mirror to the concave grating, it is easy to image it placed on the Rowland circle with its normal directed toward the grating as shown in fig. 10. The primary image of a spectral line focussed in the plane of the mirror will now act as a slit or source of light, its position being defined by the general equation of the grating

$$m\lambda = \sigma(\sin\alpha + \sin\beta_m).$$

Its secondary spectrum along the Rowland circle is defined by

$$k\lambda = \sigma(\sin\beta_m + \sin\gamma_k).$$

Considering the passage of light in fig. 10, we see that the final order of this spectrum, owing to the reflection at the mirror, is $k - m$. Consequently, the angular dispersion and resolution are here represented by

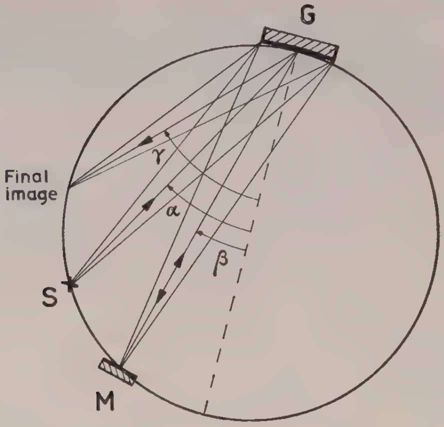

Fig. 10.

$$\frac{d\gamma}{d\lambda} = \frac{k - m}{\sigma \cos \gamma_k} \tag{5 a}$$

and

$$\frac{\lambda}{d\lambda} = (k - m) R, \tag{5 b}$$

where R is the resolving power of the grating in its 1st order.

For $k = m$, corresponding to a beam of rays diffracted secondarily to the slit, the dispersion is zero. This is immediately realized from the fact that all primary lines, λ_m, reflected at the mirror, will coincide as secondary lines for $k = m$ at the slit. Regarding our expression (5 b) for the resolution in the secondary spectrum, this is in accordance with the general picture of differences in the paths of the bordering rays of the diffracted beams of light, if we consider that these rays are reversed by the intermediate reflection at the plane mirror.

§. 6. The concave grating and the plane mirror in a parallel beam of light and by stigmatic configuration

A cooperation instead of the counteraction between both stages of diffractions in 5 a, b may be attained if the plane mirror of fig. 10 is replaced by a concave or a convex mirror having its centre of curvature in the primary line to be considered. The astigmatism of the concave grating along its Rowland circle, however, makes such an arrangement less well adapted to our case.

A simple solution of this difficulty is attained by forming the diffracted rays into a parallel beam of light, especially considering the opportunities thus afforded of producing stigmatic reproduction of the slit in the secondary spectrum. This is readily made clear by proceeding from the general equation for image formation at the concave grating:

$$\frac{\cos\alpha}{\varrho}+\frac{\cos\beta}{\varrho}-\frac{\cos^2\alpha}{a}-\frac{\cos^2\beta}{b}=0, \qquad (6\,\text{a})$$

where ϱ is the radius of curvature for the grating, a and b the distances of the light source (i.e. the slit), and its spectral image from the apex of the grating respectively. Our condition that the diffracted light of wave-length λ shall form a parallel beam, i.e. $b=\infty$, includes one of the conditions for stigmatic configuration, namely $b=\infty$ and $\alpha=0$. Another condition for stigmatic reproduction, $a=\infty$, $\beta=0$, corresponds to a parallel beam diffracted in the direction of the grating-normal (Ref. 6). Both of these conditions are fulfilled if we place a plane mirror as shown in fig. 11.

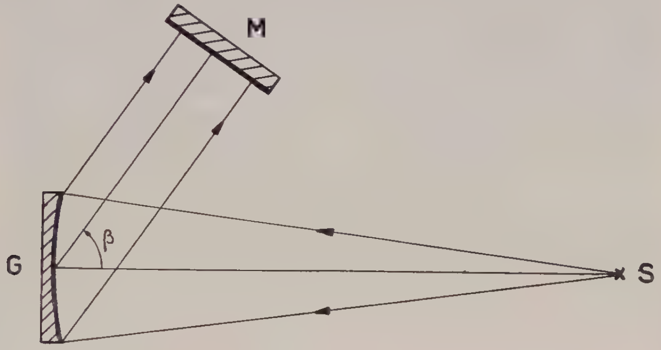

Fig. 11.

According to the first condition, the grating-normal is directed toward the slit, placed at a distance

$$a=\frac{\varrho}{1+\cos\beta}. \qquad (6\,\text{b})$$

The parallel beam, diffracted at the angle β from the grating G, is reflected normally by the plane mirror M and once more diffracted at the grating, now in accordance with the second condition $a=\infty$, $\beta=0$. Again, we may divide the passage of light into two stages:

slit–grating–mirror,	mirror–grating–slit,
$a=\dfrac{\varrho}{1+\cos\beta}$,	$a'=\infty$,
$\alpha=0$,	$\alpha'=\beta$,
$b=\infty$,	$b'=a$,
$\beta=\arcsin\dfrac{m\lambda}{\sigma}$,	$\beta'=0$.

Obviously, the secondary spectrum is of the order $2m$ and autocollimated around the slit. All other orders of λ, diffracted by the grating, e.g. $m\pm1$, are highly convergent or divergent and therefore they do not come to focus at the slit after reflection at the mirror. This is still more the case with the rays of $m\lambda$ suffering multiple reflections and diffractions at the mirror and the grating, converging repeatedly at the surface of the grating. The final images of these spectra are located in the space between the slit and the grating, some of them very brilliant although of low dispersion.

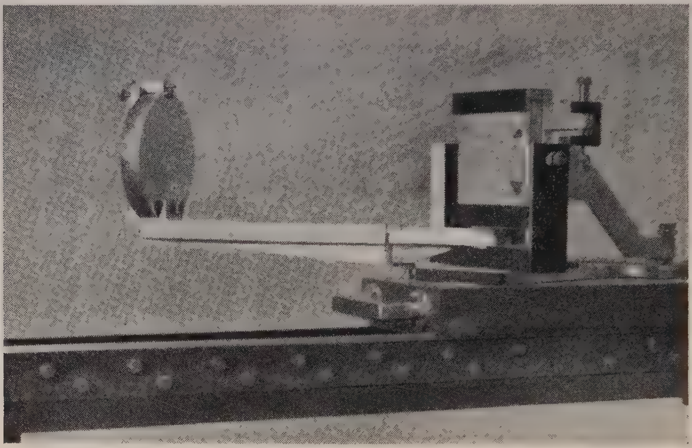

Fig. 12.

Fig. 12 shows the mounting of the concave grating and plane mirror that was used in our investigation — a heavy carriage, sliding on an optical bench of 7 meters in length. The grating was ruled on speculum metal in Rowland's engine, 1889, ($\varrho = 640$ cm, $\sigma = 12\,500$ Å or 20 000 lines/inch, ruled surface 55×138 mm^2). The mirror is made of aluminized glass, 160 mm in diameter, its planeness estimated at $^1/_4\,\lambda$ of green light. The grating and the mirror are rotatable about vertical axes in the centre line of the grating, similarly to the plane grating mounting described in § 1.

In order to estimate the brilliancy of the grating in this mounting a series of photographes are given in fig. 13. In 13 a and b the Hg- line λ 4358 is photographed in its 4th order in the stigmatic mounting and in the Eagle mounting. The exposures were performed in similar conditions in respect of slit-width ($1.5 \cdot 10^{-2}$ mm) photographic plates (Ilford thin film, half tone) taken from the same package, developer and so on. Judging from the times of exposure, 15 minutes in the stigmatic mounting and 8 hours in the Eagle mounting, the gain in brilliancy of this new mounting is very high. In this connection the astigmatism, so unfavourable for the Eaglemounting, means a great deal. It is, on the other hand, unnecessary to point out that the superiority of the stigmatic mounting would appear still more exaggerated if one used a grating of marked blaze in its second order of spectrum. The complete absence of blaze at our grating was demonstrated by the fact that the Hg-line λ 4358 appeared with equal intensity when the mirror was turned over into the opposite side of the grating.

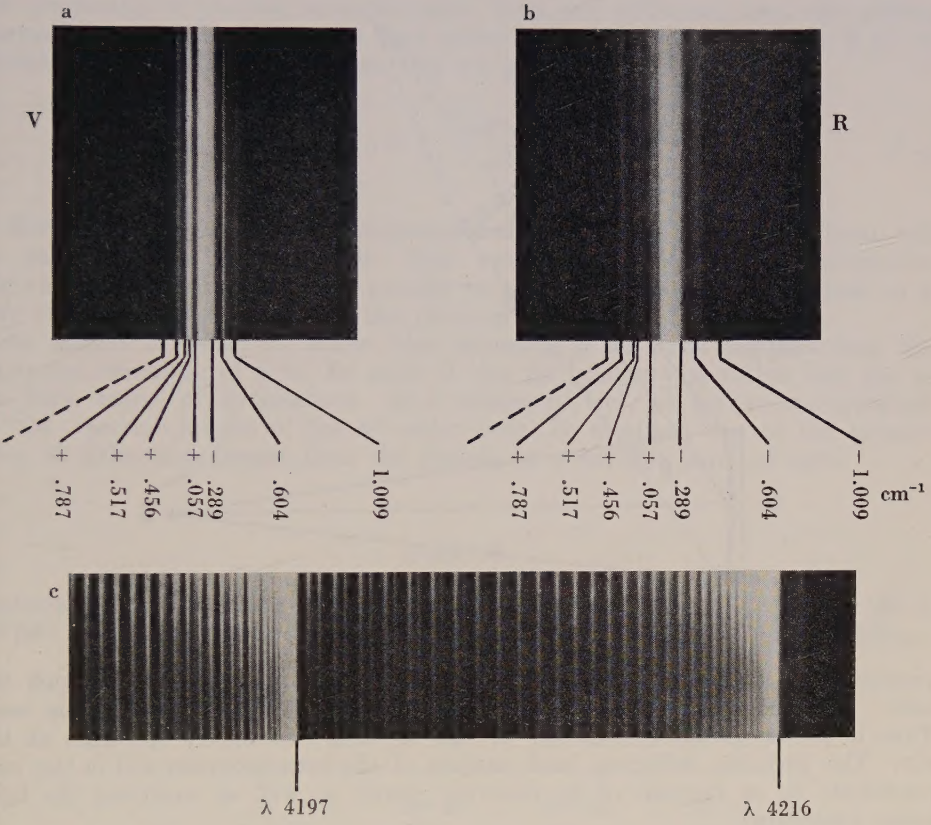

Fig. 13.

13 a, b. Hg λ 4358 in the 4th order of the stigmatic and of the Eagle mountings. The hyperfine structure components are given according to Schüler and Jones (Ref. 7). Regarding the violet component (dotted) see Nagaoka and Mishima (Ref. 8) and Wood (Ref. 9). 13 b is less exposed than 13 a.

13 c. The CN band λ 4216 in the 4th order of the stigmatic mounting.

As an additional demonstration of the brilliancy of the grating in the stigmatic mounting the CN-bands at λ 4216 of 10 minutes exposure on Ilford thin film, half tone are shown in 4th order in fig. 13 c.

The stigmatic configuration in the grating-mirror mounting makes it useful in many other respects. Its simplicity as compared with the Wadsworth mounting makes it well adapted to astrophysical investigations, e.g. as spectroheliograph. Further, a Fabry-Perot étalon, placed in the monochromatic and parallel beam of light diffracted from the grating, will form an instrument for producing "crossed" spectra of great simplicity and effectiveness. The plane mirror is here superfluous, as the spectrum may be configurated either directly by a lens L, as in fig. 14, or reflected *perfectly* by the Fabry-Perot, finally focussed as a "crossed" secondary spectrum near the slit. In the first case mentioned the Fabry-Perot acts in both directions, transmitting and reflecting the beam com-

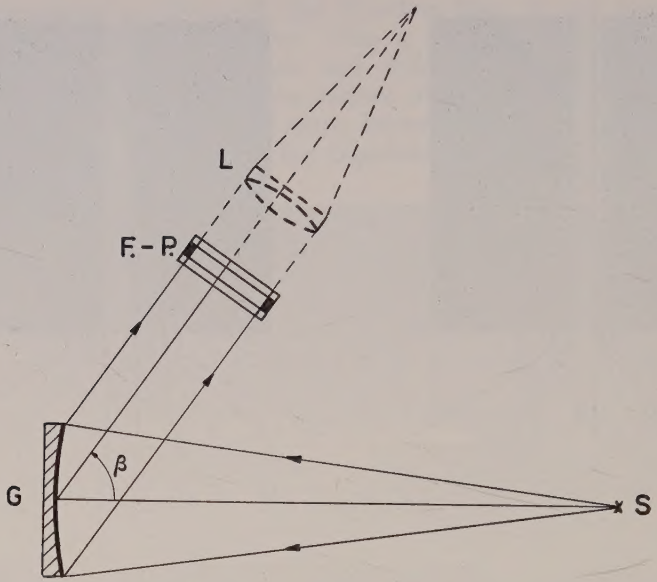

Fig. 14.

plementarily, although observations are performed only forwards through the lens. The second case is applicable, provided that electric registration is used. Thus it perserves autocollimation of the "crossed" secondary spectrum at the slit. The perfectly reflecting back surface of the interferometer will in this case contribute to an increase of its resolving power, as well as rectifying the light beam backwards.

§ 7. The concave grating and the plane mirror in parallel rays of light and by astigmatic configuration

As a parallel case to that discussed in connection with the plane grating in § 3, we will here analyse the conditions when the optical axis of the concave grating is directed toward the plane mirror, as in fig. 15. The conditions

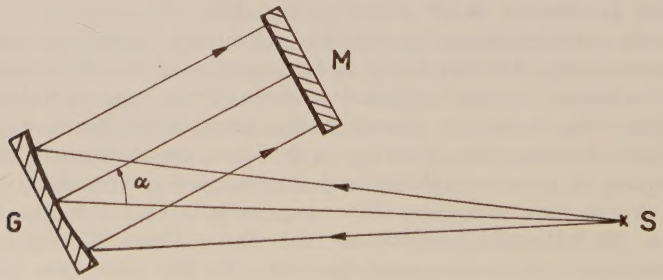

Fig. 15.

for producing a parallel monochromatic beam $m\lambda$ diffracted from the grating normally toward the mirror are once more derived from the equation (6 a) by introducing $b = \infty$, $\beta = 0$. From this we get

$$a = \varrho \frac{\cos^2 \alpha}{1 + \cos \alpha}. \tag{7 a}$$

Here, as in the case of the stigmatic configuration, § 6, the $2m\lambda$ beam will be autocollimated near the slit. This configuration, however, is astigmatic. The adjustment of the mirror parallel to the grating is easily performed in a way similar to that applied to the plane grating (§ 3).

As shown in table II below, this mounting is far more compact than the stigmatic mounting of § 6. In spite of this its brilliancy is rather low, due to its high degree of astigmatism. As a matter of fact, no perfect configuration of the spectral images of the 4th order could be attained, due to the appearance of diffraction fringes from the grating in a parallel beam of light.

Table II

Distance slit–grating for a concave grating with $\varrho = 640$ cm (21′) and $\sigma = 12\,500$ Å (20 000 lines to the inch) mounted at normal incidence ($\alpha = 0$), normal diffraction ($\beta = 0$) and for $\alpha = \beta$.

α, β	$2m\lambda$	$\alpha = 0$	$\beta = 0$	$\alpha = \beta$
0°	0 Å	320 cm	320 cm	320 cm
15°	6 475	326	304	309
30°	12 500	343	257	277
45°	17 675	375	187	226
60°	21 650	427	107	160
75°	24 225	512	35	83
90°	25 000	640	0	0

§ 8. The grating and the mirror in parallel rays of light at equal angles of incidence and diffraction

As an intermediate between the two cases of normal incidence, $\alpha = 0$, and normal diffraction, $\beta = 0$, discussed above, it seems favourable to consider the case of equal angles of incidence and diffraction, $\alpha = \beta$, especially for gratings having a strong blaze.

In the case of a plane grating mounted in autocollimation the arrangement is simply to use a telescope lens with a high-reflecting (~ 90 %) plane back surface toward the grating. Repeated diffractions and reflections of the light between the grating and the lens will increase its dispersion and resolution.

Applying the condition $\alpha = \beta$ to a concave grating we get a parallel beam of diffraction if $a = \frac{\varrho}{2} \cos \alpha$. If we place the plane mirror near the slit with its normal directed toward the grating as shown in fig. 16, it will reflect the beam to the grating for a second diffraction.

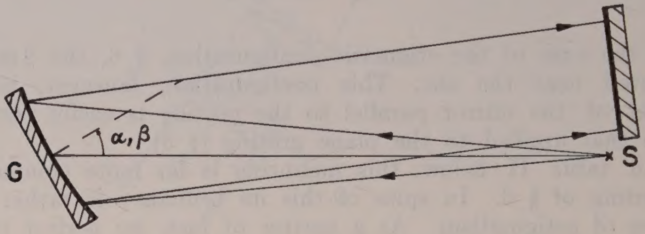

Fig. 16.

The secondary spectrum will then appear in autocollimation near the slit. This mounting has some advantages. The mirror is kept in a fixed position and the astigmatism of the configuration is favourable as compared with that of § 7.

The interferometer described in § 6 and the mountings here suggested will be discussed in a later publication.

March 1950, Physics Department, The University, Stockholm.

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